Contents

Thm has a name

Chapter 7

Theorem 1.1.1 (Dimension Thm)**.** If A is an $m \times n$ matrix, then rank$A$ + dim Null $(A) = n$.

Chapter 8

Theorem 1.2.1 (Rank-Nullity Thm)**.** Let $\mathbb{V}$ be an n-dimensional vector space and let $\mathbb{W}$ be a vector space. If $L : \mathbb{V} \to \mathbb{W}$ is linear, then

$$\text{rank}(L) + \text{nullity}(L) = n$$

Chapter 9

Theorem 1.3.1 (Gram-Schmidt Orthogonalization Thm)**.** Let $\{\vec{w}_1, \ldots, \vec{w}_n\}$ be a basis for an inner product space $\mathbb{W}$. If we define $\vec{v}_1, \ldots, \vec{v}_n$ successively as follows:

$$\begin{aligned}
\vec{v}_1 &= \vec{w}_1 \\
\vec{v}_2 &= \vec{w}_2 - \frac{\langle \vec{w}_2, \vec{v}_1 \rangle}{||\vec{v}_1||^2} \vec{v}_1 \\
\vec{v}_i &= \vec{w}_i - \frac{\langle \vec{w}_i, \vec{v}_1 \rangle}{||\vec{v}_1||^2} \vec{v}_1 - \cdots - \frac{\langle \vec{w}_i, \vec{v}_{i-1} \rangle}{||\vec{v}_{i-1}||^2} \vec{v}_{i-1}
\end{aligned}$$

Theorem 1.3.2 (QR-Decomposition)**.** I just type this out since it has a name...

Theorem 1.3.3 (The Fundamental Theorem of Linear Algebra)**.** If A is an $m \times n$ matrix, then $\text{Col}(A)^\perp = \text{Null}(A^T)$ and $\text{Row}(A)^\perp = \text{Null}(A)$. In particular,

$$\mathbb{R}^n = \text{Row}(A) \oplus \text{Null}(A) \quad \text{and} \quad \mathbb{R}^m = \text{Col}(A) \oplus \text{Null}(A^T)$$

Theorem 1.3.4 (Approximation Thm)**.** Let $\mathbb{W}$ be a finite dimensional subspace if an inner product space $\mathbb{V}$. If $\vec{v} \in \mathbb{V}$, then the vector closest to $\vec{v}$ in $\mathbb{W}$ is $\text{proj}_{\mathbb{W}}(\vec{v})$. That is,

$$||\vec{v} - \text{proj}_{\mathbb{W}}(\vec{v})|| < ||\vec{v} - \vec{w}||$$

For all $\vec{w} \in \mathbb{W}, \vec{w} \neq \text{proj}_{\mathbb{W}}(\vec{v})$.

Chapter 10

Theorem 1.4.1 (Triangularization Thm)**.** If A is an $n \times n$ matrix with real eigenvalues, then A is orthogonally similar to an upper triangular matrix T.

Theorem 1.4.2 (Principal Axis Thm)**.** Every symmetric matrix A is orthogonally diagonalizable.

Chapter 11

Theorem 1.5.1 (Schur's Thm)**.** If $A \in M_{n \times n}(\mathbb{C})$, then A is unitarily similar to an upper triangular matrix T. Moreover, the diagonal entries of T are the eigenvalues of A.

Theorem 1.5.2 (Spectral Theorem for Hermitian Matrices)**.** If A is Hermitian, then A is unitarily diagonalizable.

Theorem 1.5.3 (Spectral Theorem for Normal Matrices)**.** A matrix A is normal if and only if it is unitarily diagonalizable.

Theorem 1.5.4 (Cayley-Hamilton Thm)**.** If $A \in M_{n \times n}(\mathbb{C})$, then A is a root of its characteristic polynomial $C(\lambda)$.