My first

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hello

Let $\mathbb{L}$ be the vector space of all linear mappings from $\mathbb{R}^2 \to \mathbb{R}^3$ under standard addition and scalar multiplication of linear mappings. Let

$$\mathbb{S} = \{L \in \mathbb{L} \mid L(1,0) = (0,0)\}$$

Given that $\mathbb{S}$ is a subspace of L, find a basis for S.

Solution: Let $L \in \mathbb{S}$ and let $[L] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, by definition, L satisfies

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = L(1,0) = [L] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$$

Hence, $a = 0$ and $c = 0$. Thus we have

$$[L] = \begin{bmatrix} 0 & b \\ 0 & d \end{bmatrix}$$

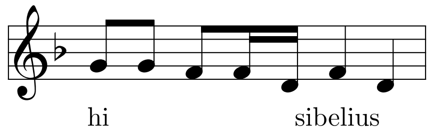
Therefore, every $L \in \mathbb{S}$ satisfies

$$L(x_1, x_2) = [L] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} bx_2 \\ dx_2 \end{bmatrix} = b \begin{bmatrix} x_2 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$$

Thus, if we define

$$L_1(x_1, x_2) = (x_2, 0), \quad \text{and} \quad L_2(x_1, x_2) = (0, x_2)$$

Then, $\{L_1, L_2\}$ spans $\mathbb{L}$ and is clearly linearly independent (since neither vector is a scalar multiple of each other) and hence is a basis.


hi
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