1.1.3.2. Final Note

很多人上完这门课，微积分就结束了，因为general cs不需要后面的微积分了。微积分相比线代对cs的作用不是那么大，但是还是看选择的cs方向。如果选择computer graphics的话，建议接着往下学微积分（math237）。cs370（Numerical Computation）是cs488(computer graphics)的preq。如果要上cs370，微积分的底子就显得尤为重要了。

很久以前cs是必修4门微积分的（amath231）[2]。后来cs考虑到要上更多的cs课，就把微积分必修的要求减轻了，只用两门了。当然从现在的水平来看，微积分确实没有很大的用处，所以先学着备着吧，假如将来真的做computer graphics，微积分的用处就显现出来了。

再说一个用处，当然这是建立在对数学很感兴趣的前提下，如今很多人对数学也就是专业需要就学一下，并没有很大兴趣。好了，进入正题。pmath365（differential geometry，微分几何）也是很有趣的一个数学subject，preq：amath231/math247，这是很需要微积分做底子的，不过更多的是从分析、理论的层面上，不是像math137/138 或者amath等等从应用的层面上。学完这个，就能感受到数学之美了[3]。还有pmath467（Algebraic Topology），preq：pmath347/351。这个偏代数，但是学起来一定是很有趣的。

还有以后如果想学习real/complex analysis, 微积分也是尤为重要的。当然如果137/138上过来，要pmath333过渡一下到pmath351/352，或者图简单，上pmath331/332也行，但是这个简单的版本偏应用，理论相对较少，所以难度也是降低了很多。

如果从amath专业来考虑的话，微积分要学好，然后real/complex analysis学baby version 就够了，多学点应用便好。相反的，pmath专业或者enthusiasts 学351/352 就比较好了，见识和学习到更多数学好玩的地方。

1.1.4. MATH 147

1.1.4.1. Topics

- The real numbers
- Sequences and limits
- Logarithms, exponentials and other important functions
- Functions, limits and continuity
- Intermediate Value and Extreme Value Theorems
- Derivatives and curve sketching
- The Mean Value Theorem and applications
- Taylor's theorem

1.1.5. MATH 237

1.1.5.1. Selected Proofs

Theorem 1.1.1

Suppose f is in C^1 at $\vec{a}$, then f is differentiable at $\vec{a}$.

Proof

▸ (for $n = 2$, but the proof is identical for any n.)
Denote the function by $f(x, y)$, and the point $\vec{a} = (a, b)$.

[2] 一个amath prof告诉我的......
[3] 我将来打算上，不知道有没有机会[捂脸]

Our hypothesis: f_x, f_y both exist near (a,b) and are continuous at (a,b)
The linear approximation is

$$L(x,y) = f(\vec{a}) + (\nabla f)(\vec{a}) \cdot (\vec{x} - \vec{a}) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

We want to show

$$\lim_{(x,y)\to(a,b)} \frac{|f(x,y) - L(x,y)|}{\sqrt{(x-a)^2 + (y-b)^2}} = 0$$

$$\begin{aligned} f(x,y) - L(x,y) &= f(x,y) - f(a,b) - f_x(a,b)(x-a) - f_y(a,b)(y-b) \\ &= \underbrace{f(x,y) - f(a,y)}_{f_x(x_0,y)(x-a) \text{ by (2)}} + \underbrace{f(a,y) - f(a,b)}_{f_y(a,y_0)(y-b) \text{ by (3)}} - f_x(a,b)(x-a) - f_y(a,b)(y-b) \end{aligned} \tag{1}$$

Consider the function

$$h(t) = f(t,y) \qquad \text{on } [a,x] \qquad\qquad (y \text{ is fixed })$$

$h'(t) = f_x(t,y)$. By hypothesis, f_x exists near (a,b), so h is differentiable on $[a.x]$ fo x close to a.

Apply MVT to this situation: there exists x_0 between a and x such that

$$h(\underbrace{x}_{t_2}) - h(\underbrace{a}_{t_1}) = h'(\underbrace{x_0}_{t_0})(\underbrace{x-a}_{t_2-t_1})$$

$$f(x,y) - f(a,y) = f_x(x_0,y)(x-a) \tag{2}$$

Consider the function

$$k(s) = f(a,s) \qquad \text{on } [y,b] \qquad\qquad (a \text{ is fixed })$$

$k'(s) = f_y(a,s)$ exists near (a,b), so k is differentiable on $[y,b]$ for y close to b.
Let $s_1 = y, s_2 = b$. Apply MVT, there exist y_0 between b and y such that

$$k(s_0) - k(s_1) = k'(y_0)(s_2 - s_1)$$

$$f(a,y) - f(a,b) = f_y(a,y_0)(y-b) \tag{3}$$

We've shown that: there exists x_0 between a and x and there exists y_0 between b and y such that

$$\frac{f(x,y) - L(x,y)}{\sqrt{(x-a)^2 + (y-b)^2}} = \frac{(x-a)[f_x(x_0,y) - f_x(a,b)]}{\sqrt{(x-a)^2 + (y-b)^2}} + \frac{(y-b)[f_y(a,y_0) - f_y(a,b)]}{\sqrt{(x-a)^2 + (y-b)^2}}$$

We want to show this $\to 0$ as $(x,y) \to (a,b)$.

$|A + B| \le |A| + |B|$ also

$$\frac{|x-a|}{\sqrt{(x-a)^2 + (y-b)^2}} \le 1 \qquad \frac{|y-b|}{\sqrt{(x-a)^2 + (y-b)^2}} \le 1$$

Then
$$\frac{f(x,y)-L(x,y)}{\sqrt{(x-a)^2+(y-b)^2}} \le \underbrace{|f_x(x_0,y)-f_x(a,b)|}_{\substack{\to 0 \text{ as } (x,y)\to(a,b) \\ \text{since } f_x \text{ is continuous} \\ \text{at } (a,b)}} + \underbrace{|f_y(a,y_0)-f_y(a,b)|}_{\substack{\to 0 \text{ as } (x,y)\to(a,b) \\ \text{since } f_y \text{ is continuous} \\ \text{at } (a,b)}}$$

So by Squeeze Theorem

$$\lim_{(x,y)\to(a,b)} \left| \frac{f(x,y)-L(x,y)}{\sqrt{(x-a)^2+(y-b)^2}} \right| = 0$$

So f is differentiable at (a,b) □

Example Prove that $\int_0^\infty e^{-x^2}\,dx$ converges, and find the value N.

Proof

▸

$$\begin{aligned} 2N &= \int_{-\infty}^{\infty} e^{-x^2}dx = \int_{-\infty}^{\infty} e^{-y^2}dy \\ 4N^2 &= \int_{-\infty}^{\infty} e^{-x^2}dx \int_{-\infty}^{\infty} e^{-y^2}dy \\ &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} e^{-x^2}dx\right]e^{-y^2}dy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dA \\ &= \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}dA \end{aligned}$$

Change to polar coordinates $\begin{cases} 0 \le r \le \infty \\ 0 \le \theta \le 2\pi \end{cases}$

$$\begin{aligned} 4N^2 &= \int_0^{2\pi}\int_0^{\infty} e^{-r^2}\underbrace{r\,dr\,d\theta}_{dA} \\ &= 2\pi\int_0^{\infty} e^{-r^2}r\,dr = 2\pi\left(-\frac{1}{2}e^{-r^2}\right)\Bigg|_0^{\infty} = \pi \end{aligned}$$

Hence $N = \dfrac{\sqrt{\pi}}{2}$ □

1.2. Algebra

1.2.1. other faculties' algebra

详尽的课程介绍可以自己去官网或者uwflow进行查找。这个课我也没上过，不过听了其他系的人上的人说也不难......

MATH 103 Introductory Algebra for Arts and Social Science

MATH 106 Applied Linear Algebra 1

MATH 114 Linear Algebra for Science